# Application of the SSOR Preconditioned CG Algorithm to the Vector FEM for 3-D Full-Wave Analysis of Electromagnetic-Field Boundary-Value Problems

Ru-Shan Chen, *Member, IEEE*, Edward Kai-Ning Yung, *Senior Member, IEEE*, Chi H. Chan, *Senior Member, IEEE*, Dao Xiang Wang, *Student Member, IEEE*, and Da Gang Fang, *Senior Member, IEEE*

***Abstract*—The symmetric successive overrelaxation (SSOR) preconditioning scheme is applied to the conjugate-gradient (CG) method for solving a large system of linear equations resulting from the use of edge-based finite-element method (FEM). For this scheme, there is no additional computing time required to construct the preconditioning matrix and it contains more global information of the coefficient matrix when compared with those of the banded-matrix preconditioning scheme. The efficient implementation of this preconditioned CG (PCG) algorithm is described in details for complex coefficient matrix. With SSOR as the preconditioner and its efficient implementation in the CG algorithm, this PCG approach can reach convergence in five times CPU time shorter than CG for several typical structures. By comparison with other preconditioned techniques, these results demonstrate that SSOR preconditioning strategy is especially effective for CG iterative method when an edge FEM is applied to solve large-scale time–harmonic electromagnetic-field problems.**

***Index Terms*—Conjugate-gradient method, finite-element method, preconditioning technique, symmetric successive over-relaxation.**

## I. Introduction

THE finite-element method (FEM) has been applied to the analysis of problems in electromagnetics for over 30 years. It can combine geometrical adaptability and material generality for modeling arbitrary geometry and materials of any composition. As a result, a large number of research papers can be found in literature [1]–[4] and some books systemically describing the application of this method in electromagnetics are currently available [5], [6]. A finite-element model is natural when the problem contains inhomogeneous material regions so that surface integral-equation (IE) methods are either incapable of modeling or are very costly to model. When applied to three-dimensional (3-D) problems in electromagnetics, the number of unknowns escalates rapidly as the size of the problem increases. Therefore, the limiting factor in dealing with 3-D problems is the unknown count and the associated demands on computer storage and solution time. Techniques that have $O(N)$ storage and solution times are, thus, necessary to tackle 3-D problems, where $N$ represents the total unknown number. This is one of the principal reasons for the popularity of partial differential equation technique over IE methods. As the problem size increases, the IE and its hybrid methods often need $O(N^l)$ storage, $1 < l \leq 2$, and quickly become unmanageable in terms of memory storage and solution time. The FEM is primarily a volume formulation for 3-D analysis and the problem domain is broken into a finite-element basis function set used to discretize the fields. The resulting linear system of equations scales as $mN$ for the FEM, where $m$ is the average number of nonzero matrix equation elements per row of the sparse linear system. This value is dependent upon the order of the finite element used, but is typically between 10–100 and is independent of the size of the mesh. For a six-unknown vector edge-based tetrahedral linear interpolation finite element, $m$ is typically less than 20 [4]. The application of the FEM to electromagnetic problems often yields a sparse, positive definite, symmetric, and very high-order system of linear algebraic equations. Solving such a system is a major computational task. The classical method includes the Gaussian elimination method and the closely related *LU* decomposition with $O(N^3)$ computational complexity. Moreover, these so-called "direct" methods bring "fill-in," i.e., nonzero entries are created in certain positions where the coefficient matrix originally has zeros. Fill-in is undesirable because it increases both the computing time and storage requirement. Although this fill-in could be reduced to some degree by selecting a suitable global ordering of the unknowns, some node-ordering strategies aim at making a coefficient matrix with a small band and present some storage schemes appropriate for such matrices. Nevertheless, these orderings are most efficient only when the mesh is "long and narrow." The best ordering strategies depend very much on the specific problem to be solved.

Direct solvers usually suffer from fill-in to an extent that these large problems cannot be solved at a reasonable cost even on the state-of-the-art parallel machines. It is, therefore, essential

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R.-S. Chen, D. X. Wang, and D. G. Fang are with the Department of Electrical Engineering, Nanjing University of Science and Technology, Nanjing 210094, China (e-mail: eerschen@mail.njust.edu.cn).

E. K.-N. Yung and C. H. Chan are with the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong.

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to employ solvers whose memory requirements are a small fraction of the storage demand of the coefficient matrix. This necessitates the use of iterative algorithms instead of direct solvers to preserve the sparsity of the finite-element matrix. Especially attractive are iterative methods that involve the coefficient matrices only in terms of matrix-vector multiplication. The most powerful iterative algorithm of these types is the conjugate-gradient (CG) algorithm for solving positive definite linear system [7]. Although the BiCG, a variation of the CG algorithm, sometimes converges much faster than the CG, its convergence is highly erratic. The convergence rate of the CG method is mainly determined by the condition number of the coefficient matrix, which is closely related to the distribution of the eigenvalues of coefficient matrix. The condition number of a linear system of equations usually increases with the number of unknowns. The solution of the linear system is the main expense of the whole problem solution, which consumes over 95% of the CPU time for FEM application in electromagnetics. It is then desirable to precondition the coefficient matrix so that the modified system is well conditioned and can converge to an exact solution in significantly fewer iteration numbers than the original system.

## II. Theory

Recently, the number of iterations required in the CG method can be controlled to some degree by the use of various preconditioning strategies [8]–[13], which attempt to transform the matrix (1) into one with more favorable properties for an iterative method as follows:

$$AX = b. \tag{1}$$

The incomplete *LU* factorizations of the coefficient matrix and its block variants are a widely used class of preconditioners [14], another is based on the factorization of the approximated inverse of the coefficient matrix [15]–[18]. However, to form these preconditioners, additional computing time is required, depending on the preconditioning algorithm. Therefore, in order to apply preconditioners to a solution of large sparse linear matrix (1) from the FEM, which has a computational complexity $O(N)$, the complexity to obtain a preconditioner should be no greater than that. The most simple and easiest way to satisfy this limit is to use a diagonal or block diagonal inverse of the coefficient matrix as a preconditioner [19], [20]. These simple preconditioners can reduce the number of iterations for dense linear systems from IE methods, but they do not result in a significant reduction of iteration for sparse linear systems from the FEM. Like the diagonal or block diagonal matrix preconditioner, the symmetric successive overrelaxation (SSOR) preconditioner can also directly be derived from the coefficient matrix without additional cost, but can lead to a significant convergence improvement for sparse linear systems. Its factorization is given *a priori*, and there is no possibility of breakdown, as in the construction phase of incomplete factorization methods [21]. It is given in a factored form and shares many properties of other factorization-based methods. Another advantage of the SSOR as a preconditioner exists in that it contains more information of the coefficient matrix when compared with a diagonal/block diagonal matrix, which is perhaps efficient only for very long and narrow structures. Therefore, the SSOR preconditioner can speed up the CG algorithm more efficiently. Although the approximate inverse preconditioning scheme is widely used for the CG algorithm [11], [18], it relies on finding a sparse preconditioning matrix $M$, which minimizes the Frobenius norm of the residual matrix. The construction of $M$ is made usually in a column-by-column manner in order to minimize the construction time. An important aspect of this approach is that only a few columns of $M$ need to be constructed, and these columns are typically chosen to refer to rows of a coefficient matrix. It can be inferred that the approximate inverse preconditioning matrix also contains more of the global information of a coefficient matrix when compared with a diagonal/block diagonal preconditioner. However, much time is spent in the construction phase and the example displayed in [18] shows that approximate inverse preconditioning scheme does not lead to a significant improvement in CPU time consumption.

In the literature [12], [13], another promising preconditioner for the CG algorithm is the algebraic multigrid method, and it reduces the overall computation time by a factor of six when compared to the SSOR scheme. However, the examples are only involved in the analysis of a two-dimensional (2-D) Poisson or scalar Helmholtz equations. The 3-D full-wave vector Helmholtz equations often met in microwave engineering are not concerned. Nevertheless, the algebraic multigrid method is a competitive preconditioner for an iterative solver and can potentially be extended to 3-D full-wave electromagnetic analysis in the future after some efforts have been taken. Although the SSOR preconditioning scheme has been developed by Saad [11], to the best knowledge of the authors, its application is not found for time–harmonic electromagnetic-field full-wave analysis in the published literature and no comparison of convergence curves is made with other preconditioning schemes. Therefore, in this paper, the SSOR scheme is applied to a vector FEM for analysis of the large 3-D Helmholtz vector-equation electromagnetic boundary-value problems.

In the SSOR preconditioning scheme, the preconditioner $M$ is chosen as follows:

$$M = (\tilde{D} + L)(\tilde{D})^{-1}(\tilde{D} + U) \tag{2}$$

where $A = L + D + U$ in (1), $L$ is the lower triangular matrix, $D$ is the positive diagonal matrix, $U$ is the upper triangular matrix, and $\tilde{D} = (1/\omega)D$, $0 < \omega < 2$. It is said that the value of $\omega$ does not have great influence on the convergence of the symmetric successive overrelaxation preconditioned conjugate-gradient (SCG) algorithm [11], and we simply choose its value to be one. Since the convergence speed of the CG method largely depends on the condition number of the coefficient matrix $A$ of (1), the linear system (1) could be scaled by a preconditioner $M$ and then transformed into the system given by

$$\hat{A}\hat{X} = \hat{b} \tag{3}$$

where

$$\begin{aligned} \hat{A} &= \tilde{D}(\tilde{D} + L)^{-1} A (\tilde{D} + U)^{-1} \\ \hat{b} &= \tilde{D}(\tilde{D} + L)^{-1} b \\ \hat{X} &= (\tilde{D} + U) X. \end{aligned} \tag{4}$$

For version A of the CG method given in [7], the straightforward preconditioned conjugate-gradient (PCG) algorithm is implemented as follows. The right-hand-side vector is initially transformed and the system is then solved for the intermediate vector $\hat{X} = (\tilde{D} + U)X$, and this vector $\hat{X}$ is multiplied by $\tilde{D}$, $(\tilde{D} + L)^{-1}$, $A$, and $(\tilde{D} + U)^{-1}$ in succession at each iterative step. When the solution converges, $X$ could be recovered from $\hat{X}$. If $NZ(A)$ denotes the number of nonzero entries in matrix $A$, this straightforward implementation of the PCG would require $6N+4NZ(A)$ multiply–adds per iteration. However, considering that $(\tilde{D}+L)$, $(\tilde{D}+U)$, the part of coefficient matrix $A$, $\hat{A}P_i$, and $\hat{A}^a R_{i+1}$ in the CG algorithm [6] can be computed efficiently. Since the efficient implementation of SCG algorithms for complex coefficient matrix is not given in [11], the SCG algorithm is given in detail as follows.

With an initial guess $\hat{X}_0$ and define

$$R_0 = \tilde{D}(\tilde{D} + L)^{-1} b - \hat{A}\bar{X}_0 \tag{5}$$

$$P_0 = G_0 = \hat{A}^a R_0 \tag{6}$$

$$K = 2\tilde{D} - D. \tag{7}$$

For $i = 0, 1, \ldots$, let

$$t_i = (\tilde{D} + U)^{-1} P_i \tag{8}$$

$$\hat{A}P_i = \tilde{D}t_i + \tilde{D}(\tilde{D} + L)^{-1}(P_i - Kt_i) \tag{9}$$

$$\alpha_i = \frac{\|G_i\|^2}{\left\|\hat{A}P_i\right\|^2} \tag{10}$$

$$\bar{X}_{i+1} = \bar{X}_i + \alpha_i P_i \tag{11}$$

$$R_{i+1} = R_i - \alpha_i \hat{A} P_i \tag{12}$$

$$W_{i+1} = (\tilde{D} + L)^{-a} \tilde{D}^a R_{i+1} \tag{13}$$

$$G_{i+1} = W_{i+1} + (\tilde{D} + U)^{-a}\left(\tilde{D}^a R_{i+1} - K^a W_{i+1}\right) \tag{14}$$

$$\beta_i = \frac{\|G_{i+1}\|^2}{\|G_i\|^2} \tag{15}$$

$$P_{i+1} = G_{i+1} + \beta_i P_i \tag{16}$$

where $\|G\| = \sqrt{\langle G, G\rangle}$ is the Euclidean norm of the vector, the inner product is defined as $\langle f, g\rangle = \sum_{i=1}^{n} f_i g_i^*$ for any two vectors $f = [f_1, f_2, \ldots, f_n]$, $g = [g_1, g_2, \ldots, g_n]$ and the adjoint matrix $A^a$ is defined by $\langle Af, g\rangle = \langle f, A^a g\rangle$ for a given matrix $A$. It can be easily inferred that $A^a = (A^*)^T$, where the asterisk denotes a complex conjugation and $T$ denotes a transposition. This efficient implementation of the SCG would require $10N + 2NZ(A)$ multiply–adds per iteration. This cost of computation is asymptotically one-half as many as the straightforward implementation if $NZ(A)$ is large enough and is almost the same as the direct CG method.

## III. Numerical Results and Discussion

Our edge-based FEM is first used to analyze a 2-D discontinuity in the waveguide filled with a full-height dielectric, as shown in Fig. 1(a). The rectangular waveguide have a width of $a = 22.86$ mm and a height of $b = 10.16$ mm and the inserted dielectric material slab has a dimension of $w = 12$ mm and $L = 6$ mm and a dielectric constant of $\varepsilon = 8.2\varepsilon_0$. In order to obtain an input reflection coefficient, one block of perfectly matched layers (PMLs) is placed at the output port to simulate the output matched load [22]. In Fig. 1(b), the curve shows our calculations of the amplitude of an input reflection coefficient $|S_{11}|$ versus frequency, which is compared with the literature [1], and an excellent agreement is found. This provides a validation of our numerical FEM method and its implementation.

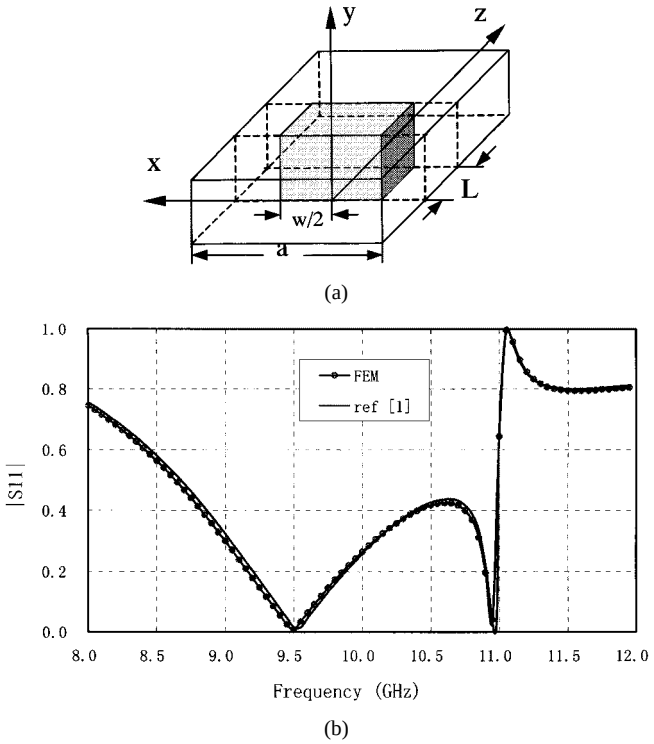


Fig. 1. (a) Configuration of a full-height dielectric-filled rectangular waveguide. (b) Magnitude of $S_{11}$ versus frequency for the dielectric post discontinuity problem with width $w = 12$ mm and length $L = 6$ mm.

Secondly, a 3-D discontinuity of a waveguide partially filled with a dielectric is shown in Fig. 2(a). The rectangular waveguide has a width of $a = 2$ and a height of $b = 1$ and the inserted dielectric material slab has dimensions of $c = 0.888$, $d = 0.399$, and $w = 0.8$ and the dielectric constant of $\varepsilon = 6\varepsilon_0$. In this edge-FEM 3-D simulation, the domain is divided into 22 400 tetrahedrons containing 5751 nodes, 30 518 edges, and 5566 forced edges. As a result, a total of 24 952 unknown edges are to be solved in a large sparse matrix equation. In Fig. 2(b), the solid curve shows our calculations of the amplitude of the input reflection coefficient $|S_{11}|$ by the SCG iterative solver. The comparison with a result from [2] is made and excellent agreement is found. This demonstrates the validation of our numerical FEM method and SCG iterative algorithm implementation.

The biconjugate gradient algorithm (BCG) is the natural extension of the CG to linear systems with the general non-Hermitian nonsingular coefficient matrices. Although it is quite competitive and sometimes converges faster than the CG approach for well-conditioned cases, the BCG iterates are not characterized by a minimization property, which means that the algorithm can exhibit a rather irregular convergence behavior with wild oscillations in the residual norm. Furthermore, the breakdown may even occur in the BCG algorithm. Fig. 3 gives the error history when the CG and BCG solvers are applied to the waveguide partially filled with a dielectric, as shown in Fig. 2,

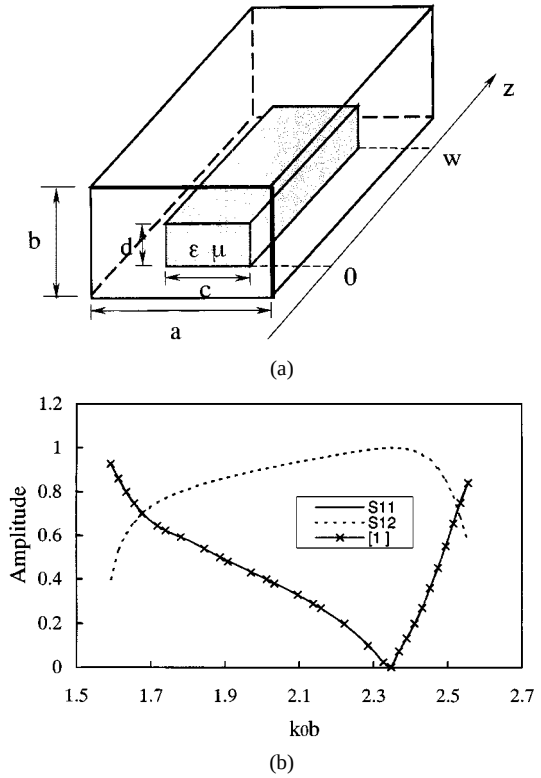


Fig. 2. (a) Configuration of a partial-height dielectric-filled rectangular waveguide. (b) Reflection and transmission characteristics of a dielectric-loaded waveguide versus normalized wavenumber.

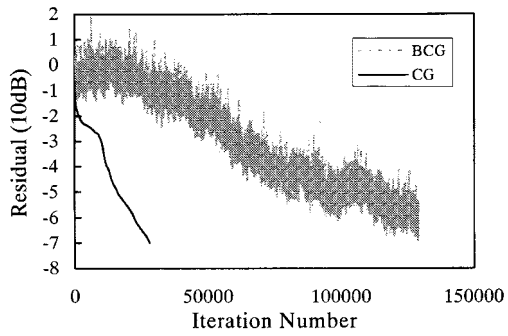


Fig. 3. Residual errors versus iteration number for the CG and BCG methods when normalized wavenumber $k_0 b = 1.88$.

where the residual errors defined as $R = ||b - Ax||/||b||$ for both CG and BCG algorithms. The CG method exhibits the monotonic convergence behavior while the BCG is rather irregular and it needs many more steps to reach convergence than the CG method because the use of PML absorbers within the computational domain significantly deteriorates the condition number of the resulting FEM system. Even though the convergence of both the CG and BCG methods can be accelerated using a preconditioning technique, which amounts to improving the condition number of the matrix, it is well known that the CG iterative solver is robust and it guarantees convergence even for a very poorly conditioned system. To investigate the efficiency of our proposed preconditioned CG algorithm for the solution of a large sparse matrix equation from the FEM, errors defined as $R = ||b - Ax||/||b||$ for both the SCG and CG versus numbers of iteration at different operating points are illustrated in Fig. 4. Therefore, the convergence behavior of the algorithms at different input reflection coefficients $\Gamma$ can be observed. Fig. 4(a)–(c) shows the convergence curves corresponding to $\Gamma = 0.02, 0.50, 0.95$, respectively. It can be found from Fig. 4 that both the SCG and CG methods converge faster for a smaller reflection coefficient case than for a larger one. This phenomenon may be attributed to the fact that the sought fields are smoother when the input matching is good and the fields vary sharply for large input mismatch. Although the total number of iterations is different for these three cases, it can be noted that the number of iteration for the SCG method is 6.22, 5.75, and 5.34 times smaller than the direct CG one to reach $-70$ dB residual errors, respectively. This computation is carried out on a Pentium 400, and in the $\Gamma = 0.02$ case, the CPU time is 3948 s for the SCG and 24 547 s for the CG algorithm. The computational time of the SCG is 6.21 times faster, nearly its improvement factor of the iteration number when compared to the CG algorithm. This fact also demonstrates that the computation cost of the efficient implementation of the SCG is almost the same as the direct CG method. In order to compare the relative speedup of the SSOR and other preconditioners, Fig. 5 displays the convergence characteristics of the SCG, tridiagonal preconditioned (TCG), diagonal preconditioned (DCG), and conventional CG algorithms. As shown in Fig. 5, the DCG algorithm has the minimal CPU cost, but only achieves 40%–50% convergence improvement. The tridiagonal preconditioner contains a bit more information of the coefficient matrix than the diagonal matrix so that the TCG algorithm converges slightly faster for the residual error to reach 55 dB. It is still not clear why it becomes a bit slower afterwards. The SSOR preconditioner possesses much more global information of the coefficient matrix $A$ than diagonal/tridiagonal matrices and it leads to better convergence improvement reasonably.

In order to investigate its adaptability to different microwave structures, the SCG solver is applied to a PML terminated microstrip line. The dielectric constant $\varepsilon_r$ is chosen to be 2.25, the operating frequency at 9 GHz and geometric dimensions are shown in Fig. 6 [18]. The domain is divided into 13 500 tetrahedrons containing 3410 nodes, 18 229 edges, and 3575 forced edges. As a result, a total of 14 654 unknown edges are to be solved in a large sparse matrix equation system. The error norms of SCG, TCG, DCG, and conventional CG algorithms are compared in Fig. 7. It can be observed that the number of iteration for the SCG method is at least four times smaller than the direct CG one when reaching $-40$ dB residual errors. Noting their respective decreasing slopes, the convergence improvement is expected to be greater if the residual errors reach $-70$ dB. In this example, the convergence improvement of the TCG is also larger than the DCG, but the difference is reduced if the smaller residual errors are required. By comparison with the results in [18], the SSOR preconditioning technique can achieve more than $2\times$ improvement in convergence than that of approximate inverse one. Since the SSOR preconditioner is related to the coefficient matrix $A$, the efficient implementation of its algorithm makes the computational complexity almost the same as the ordinary CG solver without preconditioning. Therefore, the CPU time saving is also almost the same as convergence

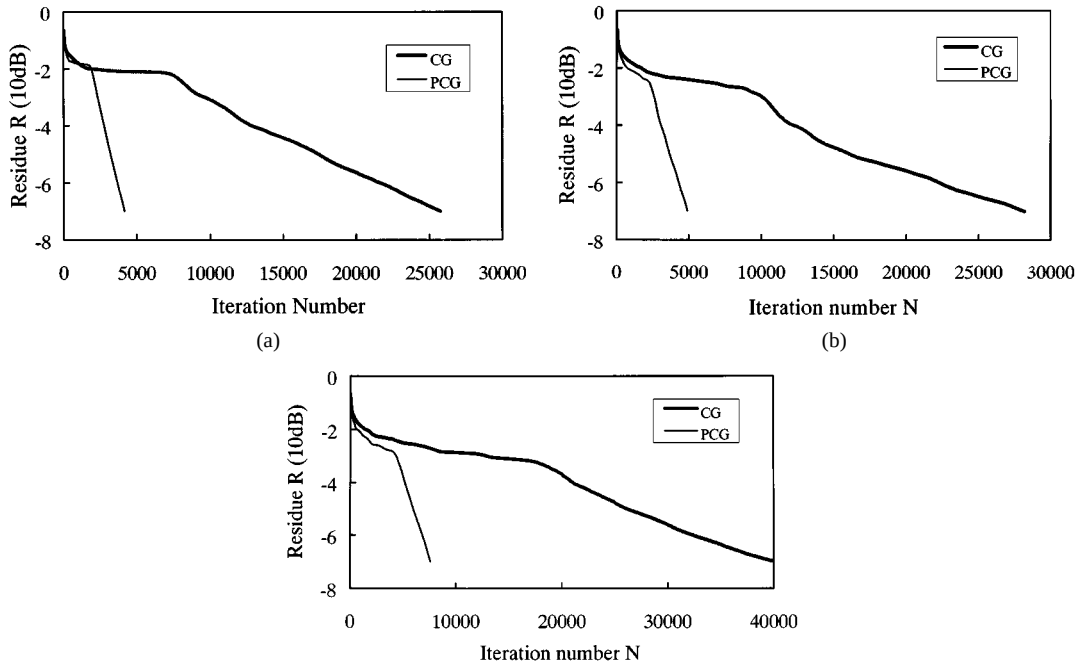


Fig. 4. (a) Residual errors versus iteration number for both the PCG and conventional CG method when normalized wavenumber $k_0 b = 2.35$ and reflection coefficient $\Gamma = 0.002$. (b) Residual errors versus iteration number for both PCG and conventional CG method when normalized wavenumber $k_0 b = 1.88$ and reflection coefficient $\Gamma = 0.50$. (c) Residual errors versus iteration number for both PCG and conventional CG method when normalized wavenumber $k_0 b = 1.60$ and reflection coefficient $\Gamma = 0.95$.

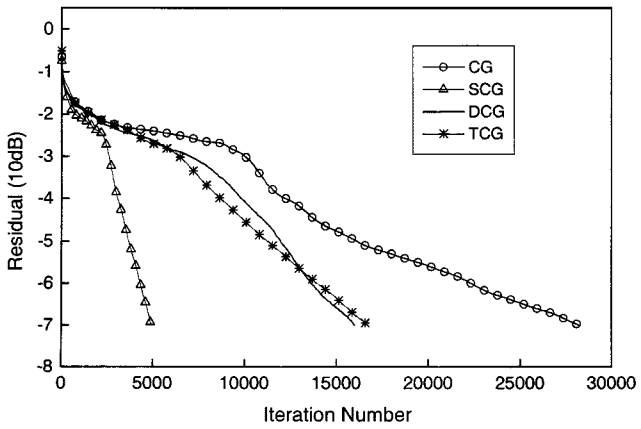


Fig. 5. Residual errors versus iteration number for SSOR, TCG, DCG, and conventional CG methods for the waveguide partially filled with dielectric when normalized wavenumber $k_0 b = 1.60$.

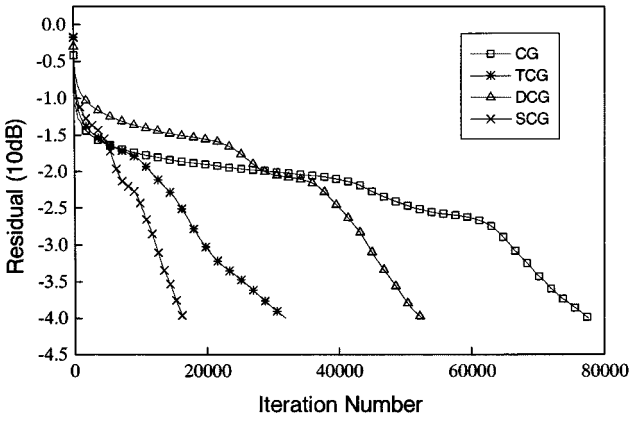


Fig. 7. Residual errors versus iteration number for SSOR, TCG, DCG, and conventional CG methods for a PML truncated microstrip line with operating frequency $\Gamma = 9$ GHz.

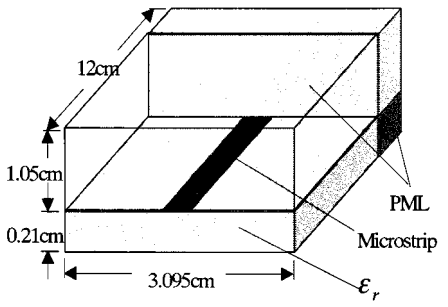


Fig. 6. Microstrip line geometry with PML truncation.

improvement. Compared to the approximate inverse preconditioning scheme, the SSOR technique has the following three advantages.

1) Its preconditioning matrix can be obtained directly from coefficient matrix $A$ without any additional computational cost and there is no possibility of breakdown as met in the construction phase of the approximate inverse matrix.
2) There is no need of additional memory to store the elements of approximate inverse matrix.
3) Since the SSOR preconditioning matrix is part of the coefficient matrix $A$, its efficient implementation in the

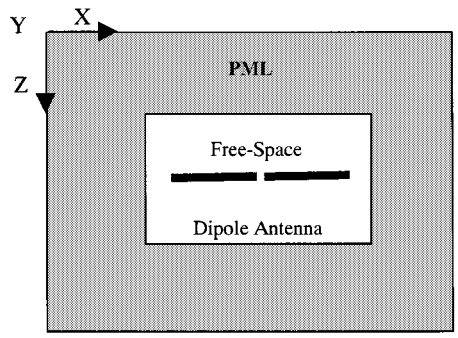


Fig. 8. Linear dipole antenna radiating in free space surrounded with PML medium and a PEC surface termination.

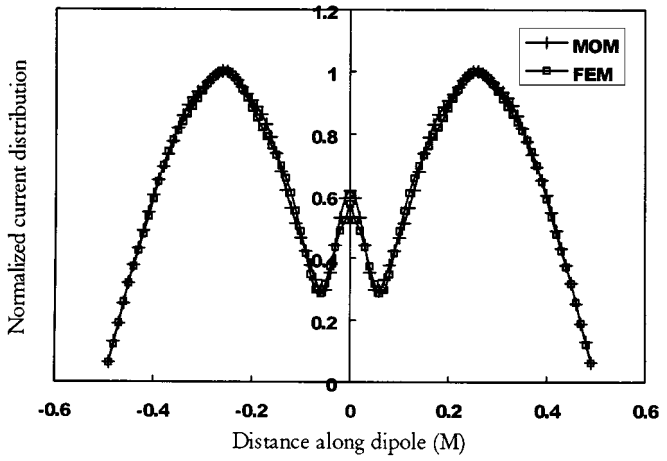


Fig. 9. Comparison of normalized current distribution along dipole antenna between the method of moments and FEM.

matrix-vector multiplication in the algorithm makes the computational cost economical and its CPU time reduces to almost one-half of the approximate inverse preconditioned solver.

The approximate inverse sparse matrix is not directly related to the coefficient matrix, therefore, its efficient implementation of the algorithm cannot be realized even though the construction of the approximate inverse preconditioner is successful.

The analysis of a linear $1\lambda$ dipole antenna radiating in free space is also taken to investigate the convergence improvement of the SCG algorithm. The 2-D frontal view of this problem geometry is given in Fig. 8. The antenna is located at the center of the problem domain and the total thickness of the PML surrounding the domain of interest is taken to be identical in all three directions. The surrounding layers of the PML are themselves terminated by perfect electric conductor (PEC) surfaces and the size of the problem domain of interest is selected as $1.8\lambda_0 \times 0.9\lambda_0 \times 0.9\lambda_0$, including the eight layers of the surrounding PML with a total thickness of $0.3\lambda_0$ in all three directions. The orthogonal FEM mesh for this domain generates a total number of 70 630 unknown. The frequency of operation is chosen to be 300 MHz. The normalized current distribution along the dipole from both the method of moments (MoM) and FEM is drawn in Fig. 9 for comparison and a good agreement is found between them. The error norms of SCG, DCG, and conventional CG algorithms are compared in Fig. 10. It can be observed that the SCG algorithm is nearly four times faster than the DCG to reach $-40$ dB residual error, while the conventional CG algorithm cannot reach the required accuracy within a reasonable time.

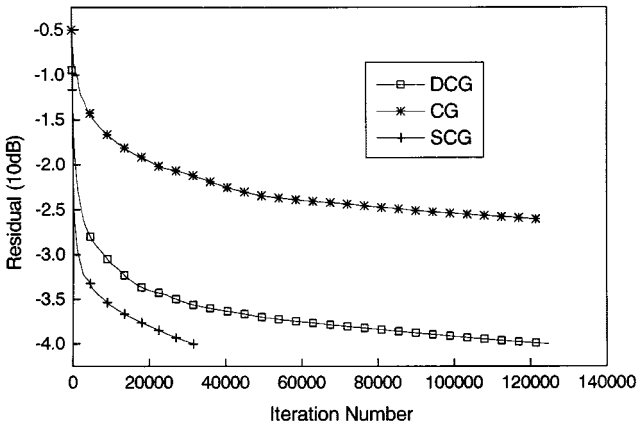


Fig. 10. Residual errors versus iteration number for SSOR, TCG, DCG, and conventional CG methods for linear dipole antenna radiating in free space with operating frequency $f = 300$ MHz.

## IV. Conclusions

In solving electromagnetic-field boundary-value problems by the FEM, the differential equation is ultimately reduced to a set of linear equations that can often be solved by iterative methods for electrically large structures. Iterative methods typically involve sparse matrix-vector multiplications, and the most popular iterative solvers are the Krylov subspace orthogonalization methods, on which CG and its variations are based. It is well known that the convergence of all iterative methods can be accelerated using various preconditioning techniques. In this paper, the SCG algorithm is proposed to solve the large sparse matrix equation obtained from the edge-based FEM. The discontinuity in a partially dielectric-filled waveguide is first analyzed as an example. Although the total iteration numbers vary at different operating frequencies, the preconditioned CG algorithm converges at least five times faster than the conventional one. When compared with the DCG and TCG algorithms, the convergence improvement of the SCG is larger since the SSOR preconditioner contains more global information of the coefficient matrix. If compared with the approximate inverse preconditioning algorithm, the SSOR does not require additional computational cost for the construction of preconditioner, and the efficient implementation of the SCG makes the computational cost at each iteration step almost one-half that for the matrix-vector multiplication, which is just the same as the conventional CG algorithm. The planar microstrip circuits and linear dipole antenna radiating in free space are also analyzed as examples to demonstrate the SCG algorithm validity for different structures. Therefore, the SCG iterative method is a powerful tool for FEM application in large time–harmonic electromagnetic boundary-value problems.

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**Ru-Shan Chen** (M'93) was born in Jiangsu, China. He received the B.Sc. and M.Sc. degrees from Southeast University, Nanjing, China, in 1987 and 1990, respectively, both in radio engineering.

Upon graduation, he joined the Department of Electrical Engineering, Nanjing University of Science and Technology (NUST), Nanjing, China, where he was initially a Teaching Assistant and then a Lecturer in 1992. In September 1996, he was a Visiting Scholar with the Department of Electronic Engineering, City University of Hong Kong, initially as a Research Associate, a Senior Research Associate in July 1997, and a Research Fellow in April 1998. From June to September 1999, he was also a Visiting Scholar at Montreal University, Montreal, QC, Canada. In September 1999, he became a Full Professor and Associate Director of the Microwave and Communication Center, NUST. He has taught several courses, including "Computer Language and Algorithm," "Professional English," "Electromagnetic Field and Wave Theory," "Computational Electromagnetics," "Microwave Ferrite Theory and Applications," and "Electromagnetic Compatibility." He has authored or co-authored over 80 papers, including 42 papers that have appeared in international journals. His research interests mainly include microwave/millimeter-wave systems, measurements, antennas, circuits, and computational electromagnetics.

Dr. Chen was the recipient of the 1992 Third-Class Science and Technology Advance Prize presented by the National Military Industry Department of China, the 1993 Third-Class Science and Technology Advance Prize presented by the National Education Committee of China, the 1996 Second-Class Science and Technology Advance Prize presented by the National Education Committee of China, and the 1999 First-Class Science and Technology Advance Prize presented by JiangSu Province. He was also the recipient of the NUST Excellent Honor Prize for academic achievement in 1994, 1996, 1997, 1999, and 2000.

**Edward Kai-Ning Yung** (M'85–SM'85) was born in Hong Kong. He received the B.Sc. degree in electrical engineering (with special distinction), and the M.Sc. and Ph.D. degrees from the University of Mississippi, University, in 1972, 1974, and 1977, respectively.

Upon graduation, he was briefly with the Electromagnetic Laboratory, University of Illinois at Urbana-Champaign. In 1978, he returned to Hong Kong and joined the Hong Kong Polytechnic. In 1984, he joined the newly established City University of Hong Kong, where he was instrumental in setting up a new department. He was promoted to Full Professor in 1989, and in 1994, he was awarded one of the first two personal chairs in the University. He is the founding Director of the Wireless Communications Research Center (formerly known as the Telecommunications Research Center). He currently heads the Department of Electronic Engineering, which is the largest of its kind in Hong Kong with 220 full-time staff members, including 65 faculties. He has authored or co-authored over 120 journal papers and has presented 140 papers in international conferences. He is the External Examiner of numerous graduate students in sister universities, both local and overseas. He is also active in applied research, consultancy and other types of technology transfers. Despite his heavy administrative load, he remains active in research in microwave devices and antenna designs for wireless communications. He is the principal investigator of many funded projects. He holds one patent. He is listed in *Who's Who in the World* and *Who's Who in the Science and Engineering in the World*.

Prof. Yung is a Fellow of the Chinese Institution of Electronics, the Institution of Electrical Engineers (IEE), U.K., and the Hong Kong Institution of Engineers. He is a member of Eta Kappa Nu, Phi Kappa Phi, Tau Beta Pi, and the Electromagnetics Academy. He is also active in professional activities. He is currently the chairman of the electronics discipline of the Hong Kong Institution of Engineers. He has been the general chairman of many international conferences held in Hong Kong. He was the president of the Hong Kong Association for the Advancement of Science and Technology (1998–1999), president of the Association of Experts for the Modernization of China (1989–1990, 1998–1999), and vice president of the Hong Kong Institution of Engineers (1999–2000). He has been the recipient of numerous awards in applied research, including the 1991 Grand Prize in the Texas Instruments Incorporated Design Championship, the 1998 Silver Medal presented at the Chinese International Invention Exposition, and the 1999 CMA Design Award. He also co-authored a paper that won the 1996 Young Scientist Award presented at the International Symposium on Antennas and Propagation, Tokyo, Japan. He holds an honorary professorship with two major universities in China.

**Chi H. Chan** (S'86–M'86–SM'00) received the Ph.D. degree in electrical engineering from the University of Illinois at Urbana-Champaign, in 1987.

From 1987 to 1989, he was a Visiting Assistant Professor at University of Illinois at Urbana-Champaign, where he was associated with the Electromagnetic Communication Laboratory. In 1989, he joined the Department of Electrical Engineering, University of Washington, Seattle, where he was an Assistant Professor. He then became an Associate Professor and achieved tenure in 1993. Since April 1996, he has been with the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong, where he is currently a Chair Professor of electronic engineering and Associate Dean (Research) of the Faculty of Science and Engineering. He is also a Guest Professor with the Xi'an Jiaotong University, Xi'an, China, and an Advisory Professor with the Nanjing University of Science and Technology, Nanjing, China. His research interests include computational electromagnetics, antenna design, frequency-selective surfaces, microstrip interconnects, and rough surface scattering. He has authored or co-authored over 200 journal and conference papers.

Prof. Chan is a Fellow of the International Commission on Illumination (CIE) and the Institution of Electrical Engineers (IEE), U.K. He was a recipient of the 1991 National Science Foundation (NSF) PYI Award. He was also the recipient of the 1998–2000 Outstanding Teacher Award (EE FT Program) presented by the City University of Hong Kong.

**Dao Xiang Wang** (S'00) was born in Jiangning, Jiangsu Province, China, in January 1976. He is currently working toward the M.Phil. degree in electrical engineering at the Nanjing University of Science and Technology (NUST), Nanjing, China

His scientific interests include microwave/millimeter-wave integrated circuit and antenna and electromagnetic numerical methods.

**Da Gang Fang** (SM'90) was born in Shanghai, China, in June 1937. He graduated from the Beijing Institute of Post and Telecommunications, Beijing, China, in 1966.

From 1980 to 1982, he was a Visiting Scholar at Laval University, Quebec, QC, Canada, and the University of Waterloo, Waterloo, ON, Canada. Since 1986, he has been a Professor at the Nanjing University of Science and Technology (NUST), Nanjing, China. In 1990, he qualified as a Ph.D. Candidate Supervisor. Since 1987, he had been a Visiting Professor with six universities in Canada and in Hong Kong. He co-edited one proceedings of an international conference, and has authored or co-authored two books, two book chapters, and nearly 300 papers, including over 30 papers appearing in international journals. He is the Vice Editor of *The Chinese Journal of Microwaves*.

Dr. Fang is a Fellow of the International Commission on Illumination (CIE). He is on the Editorial Board of the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES.